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Effect of Ring Out-of-Plane Bending Stiffness on Thermal Buckling Prediction for Ring-Stiffened Cylinders

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A NUMBER of papers have been written on the thermal buckling of ring-stiffened cylinders in which the rings are treated as discrete.[1–4] The analyses were motivated by problems encountered in space shuttle and supersonic and hypersonic aircraft design. These vehicles are subjected to thermal transients such that the thin skin heats up while the more massive rings remain cold. Radial expansion of the skin is thus prevented in the neighborhood of the rings, giving rise to rapidly varying hoop compressive stresses there. It has been shown both theoretically[1–4] and experimentally[2] that buckling of the hot skin can occur in these narrow boundary-layer regions adjacent to the cool rings. Because of the narrowness of the region under hoop compression, buckling occurs with many waves around the circumference.

Analyses of this problem to date have included rings as discrete and eccentric (ring centroid not on shell reference surface). The rings have been modeled as "line" structures with certain extensional and bending properties for deformations in the plane of the rings. (Z,θ directions in Fig. 1.) The effect on buckling loads of ring out-of-plane bending stiffness (see Fig. 1) has generally been neglected.

The purpose of this Note is to demonstrate the importance of including out-of-plane bending stiffness of the ring in thermal buckling analyses. Comparisons are made for three configurations analyzed by Chang and Card.[3] These configurations are shown in Figs. 2 and 3. Figures 4–6 show theoretical results obtained from the BOSOR3 computer program[5] in which the "out-of-plane" moments I_x and I_{xx} are included in one case and neglected in the other. Torsional stiffness and ring cross section warping are neglected in all cases. Half of the shells are analyzed, with symmetry conditions in the prebuckling analysis and antisymmetry conditions in the buckling analysis being imposed at the symmetry plane.

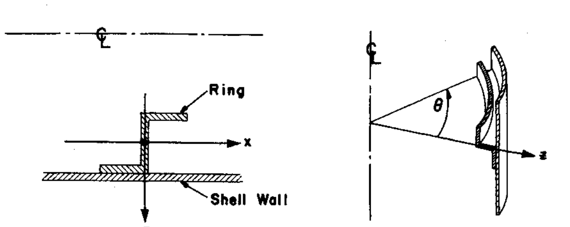

Fig. 1 Section of cylinder with internal ring. Out-of-plane bending refers to motion in x direction which varies harmonically around the circumference but involves no warping of the ring cross section.

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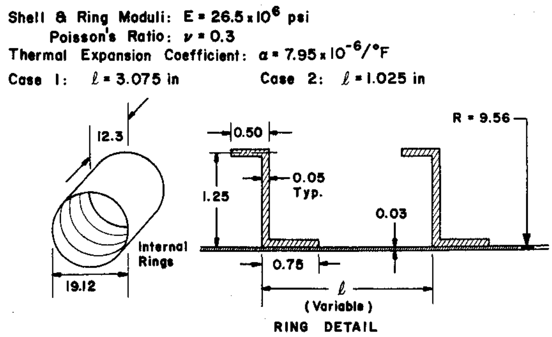

Fig. 2 Geometry of ring-stiffened cylinders, cases 1 and 2. Dimensions are in inches.

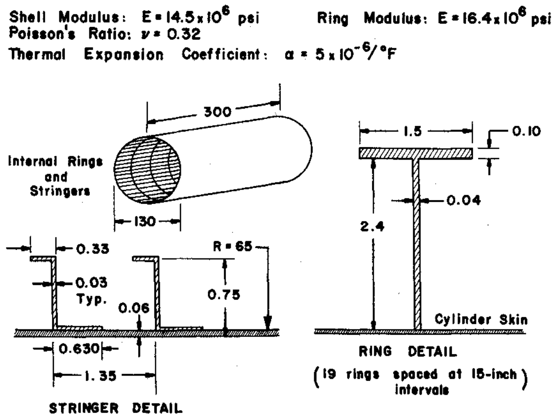

Fig. 3 Geometry of ring- and stringer-stiffened cylinder, case 3. Dimensions in inches.

Ninety-seven finite-difference mesh points are used in the analysis corresponding to Figs. 2 and 4. The shell in Fig. 3 is analyzed as consisting of two segments with 97 points in the first segment and 91 points in the second.

The most important effect is from the ring out-of-plane moment of inertia I_x. As seen from Eqs. (15), (17c), and

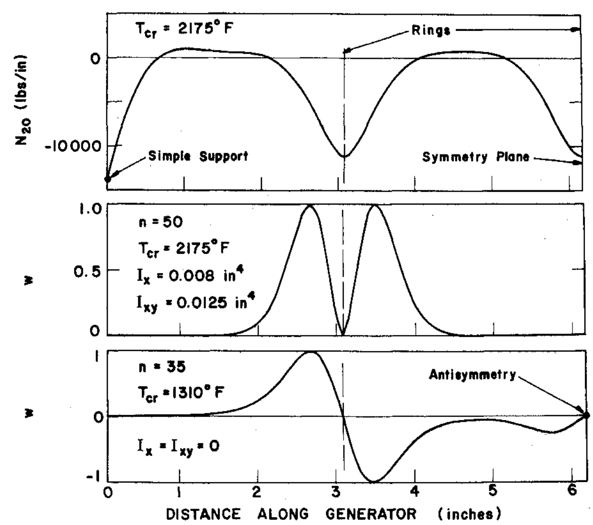

Fig. 4 Prebuckling hoop stress resultant N_{20}, buckling temperatures T_{cr}, circumferential waves n, and modes for case 1 (ring spacing = 3.075 in., see Fig. 2).

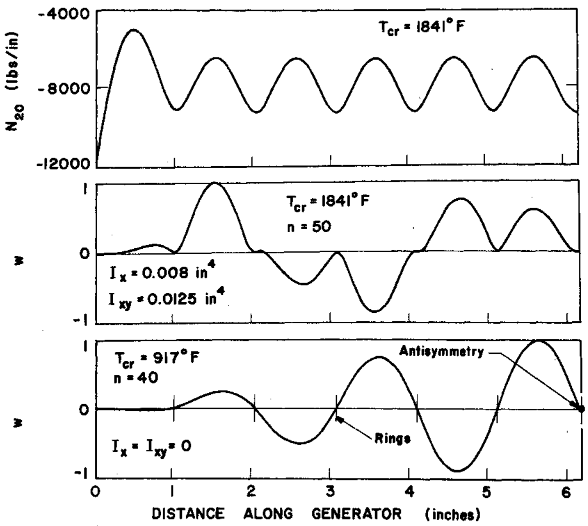

Fig. 5 Prebuckling hoop stress resultant N_{20}, buckling temperatures T_{cr}, circumferential waves n, and modes for case 2 (ring spacing = 1.025 in., see Fig. 2).

(17e) of Ref. 5, this moment of inertia is contained in a term of the ring strain energy which is multiplied by the fourth power of the circumferential wave number n. In these cases $35 \leq n \leq 75$, so that this term represents a significant contribution to the total strain energy of the system.

Note from Figs. 4 and 5 that if I_z is neglected, buckling is antisymmetric about the rings; and if I_z is accounted for, buckling is symmetric about the rings. The second term in Eq. (17c) of Ref. 5 causes this difference in predicted behavior. Additional of the effect of I_z essentially leads to a clamped condition at the ring stations. It is emphasized that this phenomenon will be less apparent if buckling is associated with a smaller number of circumferential waves.

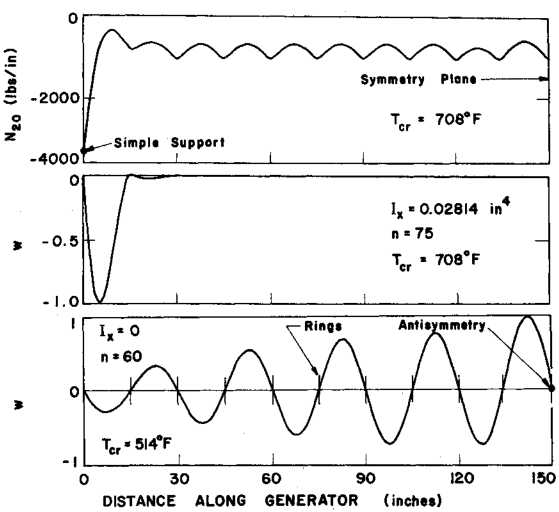

Fig. 6 Prebuckling hoop stress resultant N_{20}, buckling temperatures T_{cr}, circumferential waves n, and modes for case 3 (see Fig. 3).

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Empirical Bayes State Estimation in Discrete Time Linear Systems

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Introduction

EMPIRICAL Bayes decision theory is used here to develop a filter set for estimating the state of a linear dynamic system which does not require any assumption about the form of the underlying state disturbance error distribution or any of its moments. The filter is applicable regardless of the form of the unknown state error distribution and has been shown to provide rather stable performance characteristics for a variety of different shaped state error distributions. Some distributional assumptions on the initial state vector are also relaxed. The mean of this initial state vector is required for starting the estimation process, but knowledge of the covariance matrix and the form of the distribution is unnecessary. Thus fewer total assumptions are required to implement the filter. Finally, the filter developed here may be combined with several existing procedures, for example as in Refs. 1 or 2, for the case where the observation error covariance matrices are unknown and are to be estimated.

Statement of the Problem

Consider the following linear discrete dynamic system

$$x_n = \phi_{n,n-1}x_{n-1} + u_{n-1} \tag{1}$$

with the linear set of observations on this system

$$y_n = H_n x_n + v_n \tag{2}$$

where n is a time index, x_n is a $r \times 1$ system state vector, y_n is a $p \times 1$ observation vector, $\phi_{n,n-1}$ is a $r \times r$ state transition matrix, and H_n is a $p \times r$ matrix relating x_n to y_n. In addition, it is assumed that 1) v_n dist. $N_p(0,R_n)$ (p-normal with mean vector 0 and covariance matrix R_n); 2) $\text{Cov}(v_n,v_m) = \delta_{nm}R_n$; 3) u_{n-1} has an unknown and unspecified distribution which remains stationary over time; 4) u_n is independent of u_m for all $n \neq m$; 5) u_n is independent of v_m for all n and m; 6) $E(x_0) = c$; and 7) $\phi_{n,n-1},H_n,R_n,c$ are known. With these assumptions, the problem is to estimate x_n from the observations $y_1,y_2, \ldots, y_n$.

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